Modeling the Magnetic Effects of Field-Aligned Currents

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In this paper, I present a newly developed model of the magnetic effects of field-aligned currents. The magnetic field of a large-scale field-aligned current system is obtained by summing the magnetic fields of a large number of finite length cylindrical current elements. The resulting magnetic field is valid inside of, near and far away from the model field-aligned currents, both near the ionosphere and in the magnetosphere. The modeling technique allows for the closure of the field-aligned currents along any specified path. In order to investigate the effects of field-aligned currents on the nightside magnetospheric magnetic field and on mapping between the night-side magnetosphere and ionosphere, I add the effects of region I and II field-aligned currents to an empirical magnetic field model. Using this model, I demonstrate the following three things: (1) reasonable amounts of field-aligned current can cause the ionospheric footprint of a point at geosynchronous orbit to shift in local time by one hour or more; (2) depending on how they are closed, field-aligned currents can cause $B_z$ in the equatorial plane, near midnight, either to increase or to decrease; (3) CPS field lines can cross the equatorial plane $10 \text{RE}$ or more further from the noon-midnight meridian than they would if field-aligned current effects were not included, a result that also depends on how the currents are closed.

INTRODUCTION

Global magnetic field models are an important tool in magnetospheric physics. Several methods are used to develop these models. One is to solve, either numerically or analytically, the equations that are thought to govern the system. Another approach is to measure the magnetic field at locations throughout the magnetosphere and to develop analytical magnetic field models that fit, as closely as possible, these measurements. A third technique is to use a simple field model that corresponds to a clearly specified current system. Models developed using these approaches are referred to as physical, empirical or ad hoc, respectively. An important use of these models is mapping, along magnetic field lines, from the ionosphere to the magnetosphere and vice versa. These mappings require a magnetic field model that is as realistic as is presently possible. Virtually all mappings are carried out using empirical models [e.g., Elphinstone et al., 1991; Pulkkinen et al., 1991] although some studies involve the use of an existing empirical model that has been modified by the addition of an ad hoc model field [e.g., Pulkkinen, 1991].

Since the development of early empirical models [e.g., Olson and Pfitzer, 1974; Mead and Fairfield, 1975], models of the Earth's magnetic field have gradually become more realistic. These improvements resulted from the use of better functions to represent the magnetic field, as well as an ever-increasing data base of in situ magnetic field measurements. Since the beginning of the 1980s, the space physics community has adopted the magnetic field models developed by Tsyganenko and Usmanov [1982] and Tsyganenko [1987;1989] and most efforts made to relate ionospheric observations to prospective source regions in the magnetosphere use one or another of Tsyganenko's models (the Tsyganenko 1987 and 1989 models are referred to herein as the T87 and T89 models, respectively). The level of sophistication of present models gives one hope that the ability to map from the magnetosphere to the ionosphere, and vice versa, will soon be part of the repertoire of space physicists. It is true, however, that even the magnetic field models presently available have deficiencies that might affect the validity of mappings. These deficiencies result from differences between the actual magnetospheric current system and those currents consistent with, via Ampere's law, the model magnetic fields.

This paper discusses how currently existing models can be improved through the inclusion of the effects of a large scale field-aligned current system. The remainder of the introduction consists of a brief review of observations of field-aligned currents and the rationale for adding the magnetic effects of field-aligned currents to the T87 model magnetic field. In the following section, a new technique for modeling the magnetic effects of field-aligned currents is presented. Finally, some effects of field-aligned currents on mappings are explored.

Large-Scale Field-Aligned Currents in the Magnetosphere

A review of observations of field-aligned currents is presented here in order to describe the system to be modeled. The best information available about the actual field-aligned current distribution comes from satellite measurements of magnetic perturbations in the top-side ionosphere. Early studies using the TRIAD satellite [e.g., Zmuda and Armstrong, 1974; Yasuhara et al., 1975; Iijima and Potemra, 1976] established that the average distribution of field-aligned currents in the top-side ionosphere has a well-defined pattern. There is field-aligned current flowing either into or out of the ionosphere throughout most of the region coincident with the visual auroral oval. Data from a TRIAD satellite pass showing a magnetic perturbation, from which the existence of these currents is inferred, is included here in Figure 1. In most local time zones there are two well-defined regions of field-aligned current, which Iijima and Potemra [1978] called the region I and region II currents. The poleward part of the auroral zone contains the region I currents, whereas the equatorward part contains the region II currents. On the dawn side, the region I currents flow downward into
the ionosphere and the region II currents flow upward out of the ionosphere. On the duskside, the directions of the currents are reversed. The classic diagram showing the statistically determined locations of the region I and region II current systems is included here as Figure 2. At TRIAD altitudes, the magnitude of the eastward perturbation of the magnetic field between the two current sheets is typically between 100 and 500 nT, although values in excess of 1000 nT are occasionally observed [Zmuda and Armstrong, 1974]. The amount of current flowing in the region I current system was generally found to be somewhat larger than that in the region II system [Yasuhara et al., 1975], although near midnight the linear current density (or total current per unit of longitudinal angle) of the two current sheets is nearly equal [Iijima and Potemra, 1978]. Typical values for the amount of current in the dawn and dusk northern hemisphere region I current systems are roughly 1.5 MA for quiet times (|AL| < 100 nT) and 2.5 MA for more active times (|AL| > 100 nT), and typical current densities range from 0.5 to 1 μA/m² [Iijima and Potemra, 1978]. Near midnight, the ionospheric closure between the region I and II currents is via north-south flowing Pederson currents [Senior et al., 1982], but the field-aligned currents are always accompanied by an electric field that drives significant Hall currents [e.g., Rostoker and Boström, 1976]. Away from midnight, it is not possible for the region I and II currents to be closed entirely by meridional currents, because of their unequal linear current densities [Kamide et al., 1976; Senior et al., 1982].

The region I and II currents observed in the top-side ionosphere are thought to be the footprint of a large scale magnetospheric field-aligned current system. It is generally assumed that there is a current generation region in the magnetosphere where, for some reason, current transverse to the magnetic field is diverted to become parallel. The region I and II currents are thought to originate near or in the low-latitude boundary layer (LLBL), and near the inner edge of the central plasma sheet (CPS), respectively, where excess charge builds up due to plasma convection [Vasyliunas, 1979].

There have been numerous attempts to use particle and field measurements to determine where, in the magnetosphere, field-aligned currents are generated. Statistical pictures of the divergence of the equatorial current [e.g., Roelof, 1989; Zanetti et al., 1991] and magnetic field data from individual satellite passes through the current sheets [e.g., Kelley et al., 1986] support the view that the region II currents are generated in the near-Earth CPS and ring current regions. Low-altitude electric field, magnetic field and particle precipitation measurements indicate that the region I currents flow on closed field lines and that the ionospheric convection reversal is either embedded within [e.g., Rostoker, 1991] or at the poleward boundary of the current layer [e.g., Smiddy et al., 1980]. This supports the view that the region I currents flow to the outer magnetosphere in the high-latitude CPS or in the plasma sheet boundary layer (PSBL) in which an electric field reversal has been observed [Orsini et al., 1984]. Currents of region I sense have been observed in the PSBL via their magnetic signature [e.g., Fairfield, 1973; Ohtani et al., 1988].

In spite of the results mentioned above, there is uncertainty about where region I currents are generated and how field-aligned currents close. For instance, Stern [1983] has shown that it is possible for large scale field-aligned currents of region I sense to be produced throughout the CPS. As well, flux conservation considerations, applied to Iijima and Potemra's [1978] results, indicate that the boundary between the night-side region I and II currents is roughly 20 R_E away from the Earth (S. Skone, private communication, 1992). The coupling, or closure, of the region I and region...
II currents through the magnetosphere is not well understood. For instance, it is not clear whether results indicating a meridional connection between the region I and region II currents [Ohtani et al., 1990] and an azimuthal connection between the dawn and dusk region II currents [e.g., Roelof, 1989; Zanetti et al., 1991] apply to part (i.e., the storm substorm expansion phase associated part), or all, of the field-aligned current circuit.

Rationale

The results presented in this paper are produced using an ad hoc model of the magnetic effects of field-aligned currents superposed on the T87 model field. In this section, it is argued that the global effects of the field-aligned currents are not included in empirical models, in general, or in the T87 model, in particular. As well, the rationale behind the decision to add the effects of field-aligned currents to the T87, rather than the T89, model field is presented.

An empirical model of the magnetospheric magnetic field is a vector function which, through adjustment of the parameters \( \lambda_i^j \), is “best fit” to a data base of magnetospheric magnetic field values:

\[
\vec{B}_{\text{model}}(\vec{R}) = \sum_{i=1}^{N} \vec{f}_i(\lambda_i^1, ..., \lambda_i^{M_i}; \vec{R})
\]

Due to the complicated nature of the magnetospheric current system, the real challenge in the development of realistic models is the selection of the functions, \( \vec{f}_i \). As discussed above, the T87 and T89 are currently the most widely used empirical models. These models are a superposition of functions used to represent the magnetic effects of the ring and cross-tail currents on a model terrestrial magnetic field. A polynomial function is included to improve the fit of the model fields to the data. The difference between the two models is in the function used to represent the magnetic effects of the cross-tail current.

Empirical models are data-based and there is some expectation that the magnetic effects of the field-aligned currents will be, at least partially, already included in these models; however, there are several reasons why the magnetic effects of the global field-aligned current system are probably not included in the T87 model. First, the averaged data set on which the model is based is unlikely to contain a realistic contribution due to the global system of field-aligned currents flowing in the magnetotail [G. Rostoker, private communication]. This is a consequence of the fact that the average magnetic field at a given location in geocentric solar-magnetospheric (gsm) coordinates determined from a data set, regardless of how the data is binned in terms of geomagnetic activity (i.e., \( Kp, AE \) etc.), will be based on data from a variety of real activity levels: thickening and thinning of the CPS as well as flapping of the magnetotail itself will lead to motion of the field-aligned current layers. Thus, for grid points near the high-latitude edge of the plasma sheet where the field-aligned currents are thought to flow, the actual field-aligned currents are as likely to flow above the observation point as below. Accordingly, over a large data set, some of the effects of the field-aligned currents will cancel out and the data points representing the averaged magnetic field values will not reflect the true size of the contributions of the field-aligned currents for any individual event. Second, the polynomial function used in the Tsyganenko models allows for a better fit to the data set used than would a model employing the model ring and tail currents alone. One way of looking at this is that the data-fitting technique will obtain values for the parameters associated with the polynomial functions in the model that are chosen in response to the difference between the sum of the best-fit model ring and tail currents and the actual average magnetospheric current system. Stating that the polynomial function models the effects of the field-aligned currents is to tacitly assume that this difference is almost entirely the average field-aligned current system. If this were demonstrably true then the statement would be reasonable. For several reasons, however, this is probably not the case. For example, Tsyganenko [1989] pointed out that the use of a “taper function” in the T87 model to produce the observed wrapping of the inner edge of the cross-tail current around the Earth led to some of the model cross-tail current diverting out of the current sheet and flowing in the \( Z_{\text{gsm}} \) direction through the region of space corresponding to the magnetospheric lobes. For this reason alone, one expects differences between the real and model cross-tail current. Third, typical scale sizes of the current systems in the Tsyganenko magnetic field model range from several to tens of Earth radii. Therefore it is not possible for the functions to adequately represent the magnetic effects of relatively thin (\( \sim 1 \text{Re} \)) current sheets, especially in the inner magnetosphere.

There is another, more subtle, reason why the global distribution of model field-aligned currents generally should be regarded as an artifact of the modeling procedure. Almost all existing magnetic field models have distributed magnetospheric current systems. In the development of all of these, there has been no attempt made to constrain the current systems so that the current is exactly transverse to the magnetic field [e.g., Olson and Pfister, 1974]. Indeed, this constraint would, in general, be unreasonable to impose. For example, the model magnetic field of Mead and Fairfield [1975] is a vector function, the components of which are second-order (at most) polynomials in the cartesian coordinates \( X_{\text{gsm}}, Y_{\text{gsm}}, \) and \( Z_{\text{gsm}} \). The model is parameterized according to dipole tilt and geomagnetic activity. The coefficients are determined by least-squares fitting to a set of magnetic field measurements. If the terrestrial magnetic field is represented by a dipole and both the dipole tilt and the model field-aligned current are constrained to be zero (i.e., require \( \vec{B} \cdot \nabla \times \vec{B} = 0 \)), then it can be shown that the only possible non-trivial form for the polynomial functions gives

\[
\vec{\nabla} \times \vec{B} = A(\rho) \hat{\phi}
\]

where \( A(\rho) \) is some scalar function of \( \rho = \sqrt{X^2 + Y^2} \) and \( \hat{\phi} \) is the unit vector in the azimuthal direction. This is a current distribution with current lines that are circles in planes of constant \( Z \) centered on the \( Z \) axis. The only intramagnetospheric model currents would be azimuthally symmetric; therefore only the magnetic effects of a ring type current could be modeled. If nonzero dipole tilt angles are allowed for and the model currents are constrained to be transverse to the model magnetic field, it can be shown that the only possible nontrivial form of the polynomial functions gives

\[
\vec{\nabla} \times \vec{B} = 0
\]
eled with the functions chosen by Mead and Fairfield [1975], there will be a field-aligned component of the model current. This is true for almost all empirical models. Exceptions are models like that of Williams and Mead [1965], where both sheet on the equatorial plane) allow for a current distribution symmetry and simplicity of the model (their model consists with zero field-aligned component everywhere. It is important to realize that such asymmetries in the magnetosphere do not demand the presence of real field-aligned currents. The key point here is that when simple functions are used to model the magnetic effects of distributed magnetospheric currents, the model currents will not, in general, be transverse to the model magnetic field. It is difficult to draw conclusions about the real field-aligned current distribution, based on such a model, because there will be model field-aligned currents regardless of whether or not the magnetic effects of field-aligned currents are actually present in the data.

In light of the argument presented above, it is interesting to look at the field-aligned component T87 model current system. According to Ampere's law, the field-aligned component of the current is

\[ J_{\parallel} = \frac{1}{\mu_0} \frac{\mathbf{B} \cdot (\nabla \times \mathbf{B})}{B} \]  

(4)

In discussing the field-aligned currents in the magnetosphere it is customary to "normalize" the values of the field-aligned current obtained from equation (4) to what they would be if the current were to flow without diversion along a flux tube from where the current value is determined to the ionospheric footpoint of the field line passing through this point:

\[ J_{\parallel}' = \frac{B_i}{B} J_{\parallel} \]  

(5)

This quantity, \( J_{\parallel}' \), is the amount of current, at the location where the field-aligned current is determined, that is contained in a flux tube having an ionospheric cross-sectional area of 1 m². Figures 3 and 4 are contour plots of this quantity on the \( Y_{gsm} = -6.73R_E \) and \( X_{gsm} = -25R_E \) surfaces. The magnetic field used here is the T87 \( K_p = 3 \) model. Figure 3 shows large scale region II sense model current poleward of large scale region I sense model current in the near-Earth region and large scale region II sense model current distributed throughout the entire cross-section of the magnetotail \( 25R_E \) from the Earth. The patterns present in these figures are not consistent with those expected, based on either an extension of accepted low-altitude field-aligned current patterns into the night-side magnetotail or observations of region I sense current in the PSBL.

The view presented here is that the global magnetic effects of the large-scale field-aligned currents are not included in the T87 (or any existing) empirical magnetic field model. Here global means all latitudes and local times from the ionosphere right out to the distant magnetosphere. That there might be spatially-limited regions where these effects could be represented by the model is not inconsistent with this view.

The T87 and T89 models are currently the most widely used empirical models. The remainder of this section discusses why the T87 model field is used as a background model in this study. Tsyganenko [1989] determined that his T87 model was ineffective for modeling the magnetic field at geostationary orbit and developed the T89 model to correct this shortcoming. A comparison of the T87 and T89 model fields with geostationary orbit data indicates that he was successful [Tsyganenko, 1989]. One consequence of the difference between the T87 and T89 models is that, in the night-side magnetotail, the T89 model magnetic field is more flared away from the noon-midnight meridian than is the T87 model field. As Donovan and Rostoker [1991] showed, this additional flare provided a more realistic description of the earthward flow of plasma in the central plasma sheet than could be obtained from the T87 model, when one examined the consequences of mapping an ionospheric equipotential

![Fig. 3. Field-aligned component of the current associated (via Ampere's law) with the T87 model magnetic field (\( K_p = 3 \) version) on the plane \( X_{gsm} = -6.73R_E \). The values are normalized to what the field aligned current would be if it was mapped, without diversion, to the ionosphere (see text). The contours are separated by 0.2\( \mu_A/m^2 \). Contours indicating positive values are dashed while the zero and negative contours are solid. In the northern (\( Z_{gsm} > 0 \)) and southern (\( Z_{gsm} < 0 \)) hemispheres, current flowing towards the Earth is indicated by positive and negative values respectively.](image)

![Fig. 4. The same as Figure 3, but showing values of field-aligned current on the plane \( X_{gsm} = -25R_E \) with a contour separation of 0.4\( \mu_A/m^2 \).](image)
pattern of Heppner and Maynard [1987] back into the night-side magnetosphere. Furthermore, examination of the expected variation of the $B_z$ component of the tail field along the noon-midnight meridian [Rostoker and Boström, 1976] reveals that the T87 model provides a better representation of the observations than does the T89 model. In particular, the T89 model $B_z$ is too small, or even negative [Donovan et al., 1992].

In comparison to the T87 model, the greater flare of the T89 model magnetic field appears to be an improvement, but the T89 model equatorial $B_z$ distribution near midnight appears to be a deficiency. Both the greater flare and the small $B_z$ values are a consequence of differences between the two models in the choice of functions used to represent the magnetic effects of the cross-tail current [Donovan et al., 1992]. There is, however, another current system that can have an appreciable effect on both the flare of the magnetic field and the distribution of $B_z$ near midnight: the large scale field-aligned currents. Two effects that these currents are expected to have in the magnetotail is to make the CPS magnetic field more flared and $B_z$ more positive near midnight [Rostoker and Boström, 1976]. In light of the points made above, it would be most reasonable to add these effects to the T87 model field.

A MODEL OF THE MAGNETIC EFFECTS OF FIELD-ALIGNED CURRENTS

A useful representation of the magnetic effects of large-scale field-aligned current systems is desired. Attention is restricted to current distributions that do not vary in time. The current continuity equation

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$$

follows directly from Maxwell’s equations. For time-independent situations the current distribution must be divergence free:

$$\nabla \cdot \mathbf{J} = 0$$

The relationship between the magnetic field and the current distribution can be determined from Maxwell’s equations. For time-independent situations, Ampere’s law follows trivially:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

Using Helmholtz’s theorem [Arfken, 1985] and working in the Coulomb Gauge [Jackson, 1975], the time-independent Maxwell equations yield the Biot-Savart law:

$$\mathbf{B}(r) = \frac{\mu_0}{4\pi} \int dV' \frac{\mathbf{J}(r') \times (r - r')}{|r - r'|^3}$$

In the development of models of the magnetic effects of distributed currents within the magnetosphere, it is customary to stipulate a current distribution and then to use the Biot-Savart law to determine the corresponding magnetic field. A simple example of this approach is the use of model wire loops to represent currents in the magnetosphere. The magnetic field of any closed circuit made of infinitesimally thin wire can be determined to arbitrary accuracy using the Biot-Savart law. The standard approach is to construct the loops out of finite length segments of infinitesimally thin straight wire. The contribution to the integral in equation (9) from current flowing in the segment of wire is in the azimuthal direction about the wire segment:

$$B_\phi(\rho, z) = g_0(\rho, z)f_0(\rho, z)$$

$$g_0(\rho, z) = \frac{\mu_0 I}{4\pi \rho}$$

$$f_0(\rho, z) = \frac{z_0 - z}{\sqrt{\rho^2 + (z_0 - z)^2}} - \frac{z_0 - z}{\sqrt{\rho^2 + (z_0 - z)^2}}$$

Here the $z$ axis is parallel to the wire segment, the wire segment has endpoints at $z = z_0$ and $z = z_1$ with $z_1 > z_0$ and a current $I$ is flowing along the segment from $z_0$ to $z_1$. The current distribution that is consistent with this magnetic field can be determined from Ampere’s law:

$$J_\rho = -\frac{1}{\mu_0} \frac{\partial B_\phi}{\partial z}$$

$$J_\phi = \frac{1}{\mu_0} \frac{\partial (\rho B_\theta)}{\partial \rho}$$

For the magnetic field given in equation (10), these give

$$J_\rho = \frac{I_\rho}{4\pi} \left( \frac{1}{(\rho^2 + (z_1 - z_0)^2)^{3/2}} - \frac{1}{(\rho^2 + (z_0 - z_1)^2)^{3/2}} \right)$$

$$J_\phi = \frac{I_\phi}{4\pi} \left( \frac{z_0 - z}{(\rho^2 + (z_0 - z_1)^2)^{3/2}} - \frac{z_1 - z}{(\rho^2 + (z_1 - z_0)^2)^{3/2}} \right)$$

$$+ \frac{2(\delta(z - z_1) - \delta(z - z_0)\delta(\rho))}{\rho}$$

where $\delta$ and $H$ are the Dirac delta and Heaviside functions, respectively. The third term in the brackets in equation (16) is the (infinite) current density of the finite wire element. The presence of the other two terms in equation (16) and the radial component is, at first glance, surprising. Applying Ampere’s law to the magnetic field given in equation (10) yields a current that is distributed throughout space and yet this magnetic field, according to our naive application of the Biot-Savart law, is that of a straight current carrying wire of finite length. The problem lies in the current distribution used in the original Biot-Savart law integration. This current has a source (at $z = z_0$) and a sink (at $z = z_1$) and therefore the current is not divergence-free and thus is not consistent with any time-independent solution of Maxwell’s equations. The Biot-Savart law, however, is the magnetic field that corresponds to a given current distribution for time independent conditions only. The integral in equation (9) is over all space and, consequently, over complete circuits. When a complete model current circuit is constructed using finite lengths of infinitesimally thin straight wire segments, provided the current on every element is the same, the current density will be zero everywhere except on the wire elements. Finite length wire segments were first used to construct models of magnetospheric current systems by Olson and Pfitzer [1974], who used this technique to model the effects of the ring current and the cross-tail current with closure around the magnetospheric boundary.
There have been several efforts to model the effects of field-aligned currents in the magnetosphere. Kaufmann and Larson [1989] developed a model of field-aligned currents in order to study how the magnetic field due to these currents might affect the mapping of electric potential distributions from the ionosphere to the magnetosphere. They used Olson and Pfitzer’s [1974] technique to model the magnetic effects of the field-aligned current system. Their model of the field-aligned current system consisted of a number of current loops, each of which was constructed from a number of finite length wire elements. The current loops were placed on a background magnetic field. It was necessary to perform several iterations, tracing new field lines upon which to put current loops, using the background magnetic field and the magnetic field of all of the loops. They found that it was generally possible, using their technique, to construct “self-consistent” configurations of these field-aligned currents. Here self-consistent is taken to mean that the added current is indeed field-aligned. Stern [1993] and Tsyganenko [1991] have been working towards a more general representation of the magnetic field caused by the large-scale field-aligned currents. Both Stern and Tsyganenko are developing analytical functions to represent the magnetic field due large sheet-like current structures, with the currents flowing in a pattern expected from observations of the ionospheric end of the current system [e.g., Iijima and Potemra, 1976]. Their approach is computationally more efficient than that of Kaufmann and Larson [1989]. As well, it is possible to use this technique to try to extract information about the distribution of field-aligned currents from a large data base of in situ magnetic field measurements. The approach of Kaufmann and Larson [1989] has the disadvantage that, because the current elements are segments of wire, the current density associated with the field-aligned current model is either zero or infinite. As well, there are singularities in the magnetic field: near the wire elements, the magnetic field becomes infinite. Their approach, however, leads to a current system that is actually field-aligned. Furthermore, it is flexible enough that the model field-aligned currents can be closed in the magnetosphere along any specified path and can be used to explore the consequences of different possible current closure paths. This is an important advantage, considering the lack of knowledge about how these currents close in reality.

The approach used in the development of the model presented here is similar to that of Kaufmann and Larson [1989]. A large number of current elements are used to construct a large-scale field-aligned current system. The current elements, however, are not infinitesimally thin wire segments with an infinite current density. Instead, elements with current densities that are volume-filling are used. It is possible to specify a current distribution and use the Biot-Savart law integral to calculate the contribution to the total magnetic field. It is, however, exceedingly difficult to evaluate this integral for all but the simplest of geometries. Current elements with geometries simple enough to allow for straightforward evaluation of the Biot-Savart law integral typically have magnetic fields with singular points. An example is the infinite magnetic field strength near the wire elements discussed above. Another possibility is to specify a magnetic field with the desired properties. The corresponding current distribution, and hence the current element, can be determined by applying Ampere’s law. As pointed out above, the Biot-Savart law is a solution to Ampere’s law and the two approaches are completely equivalent. The goal here is to use a cylindrical current element with an azimuthal magnetic field. The current density is to be finite everywhere, aligned with the axis of the cylinder inside of the cylinder, and small, or zero, outside of the cylinder. As well, the element is to be of finite length. For illustrative purposes, the symmetry axis of the cylindrical element is taken to be the z axis, where \( z_0 \) and \( z_1 \) are the locations of the two ends of the element; the plane perpendicular to the z direction and located midway between \( z_0 \) and \( z_1 \) is called the midplane of the element. The element has a characteristic radius “a.” The magnetic field associated with the current element is directed in the azimuthal \((\phi)\) direction and is azimuthally symmetric:

\[
B_\phi = F(\rho, z)
\]  

(17)

Any function of this form is divergence-free. The magnetic field due the current element is of the same form as that due to a finite length of current-carrying wire:

\[
B_\phi = g(\rho, z)f(\rho, z)
\]  

(18)

For our current elements, the function \( g(\rho, z) \) will be the magnetic field of a current-carrying cylinder of radius \( a \) and infinite length centred on the z axis. The function \( f(\rho, z) \) will be an envelope function, limiting the extent of the effects of the current element in the z direction. Below, two different types of current elements that can be used to build up a model of the field-aligned currents are discussed.

The magnetic field of the first type of current element (herein referred to as a type I element) is

\[
B_\phi = g_1(\rho, z)f_1(\rho, z)
\]  

(19)

where

\[
g_1(\rho, z) = \alpha \begin{cases} 
1/\rho & \rho > a \\
1/\rho^2 & \rho < a
\end{cases}
\]  

(20)

and

\[
f_1(\rho, z) = \frac{1}{2} \left( \frac{(z - z_0)}{\sqrt{\rho^2 + (z - z_0)^2}} - \frac{(z - z_1)}{\sqrt{\rho^2 + (z - z_1)^2}} \right)
\]  

(21)

This magnetic field is the same as that of the finite length of current-carrying wire except for the radial dependence of the function \( g \). For values of \( \rho > a \), the function is the same as the magnetic field in equation (10), and hence the current associated with this element for \( \rho > a \) is the same as the current given in equations (15) and (16) with \( I \) replaced by \( 2\pi a/\mu_0 \). According to Ampere’s law, the current density associated with this current element for \( \rho < a \) is

\[
J_\rho = \frac{a\rho^3}{2\mu_0} \left( \frac{1}{(\rho^2 + (z_1 - z)^2)^{3/2}} - \frac{1}{(\rho^2 + (z_0 - z)^2)^{3/2}} \right)
\]  

(22)
This is the same as the current density in equations (15) and (16), except that the term describing the line current on the z axis in the expression for \(J_z\) has been replaced by the third and fourth terms given in equation (23). The magnetic field associated with this element is finite everywhere, as is the current density. Figure 5 is a plot of \(f_1(p, z)\). Figure 6 is a three dimensional plot of current and magnetic field line tracings using for a type I current element.

The magnetic field of the second type of current element (herein referred to as a type II element) is

\[
B = g_2(p, z) f_2(p, z)
\]

where

\[
g_2(p, z) = \alpha \left\{ \begin{array}{ll}
\frac{a^{(\gamma-1)/\rho}}{\rho^2} & \rho > a \\
\frac{a^2}{\rho^2} & \rho < a
\end{array} \right.
\]

and

\[
f_2(p, z) = \frac{1}{2} \left[ \tanh[D_0(z - z_0)] - \tanh[D_1(z - z_1)] \right]
\]

Here \(\gamma\) is a parameter normally set to 1. With Ampere's law, the current density that corresponds to this magnetic field can be determined. Setting \(a = \text{const}\) and \(D = D_0 = D_1\), this gives

\[
J_x = \frac{\alpha}{2\mu_0} \left[ \text{sech}^2[D(z - z_0)] - \text{sech}^2[D(z - z_1)] \right]
\times \left\{ \begin{array}{ll}
1/\rho & \rho > a \\
\rho/a^2 & \rho < a
\end{array} \right.
\]

\[
J_z = \frac{\alpha}{2\mu_0a^2} \left[ \tanh[D(z - z_0)] - \tanh[D(z - z_1)] \right]
\times \left\{ \begin{array}{ll}
(1 - \gamma)(a/\rho)^{\gamma+1} & \rho > a \\
1 & \rho < a
\end{array} \right.
\]

Figure 6 illustrates that the current flowing along the axis of a type I current element diverges radially away from the axis beyond the "end" of the element and returns, outside of the element (\(p > a\)), to flow back into the other end of the element. The current field lines are circles centred on the element's axis. The magnetic field current pattern here is like that of a solenoid, but with the current and magnetic field transposed. For a type II element, the magnetic field lines will again be axisymmetric circles; however, the pattern of the current flow, after it diverges away from the "end" of the cylinder, depends on the selection of the parameter \(\gamma\). Figure 7 shows that, depending on the value of \(\gamma\), the current will either flow towards larger \(z\), radially away from the cylinder's axis, or return to the other "end" of the element.

The following example helps to illustrate the rationale behind the choice of functions describing the current elements. Consider \(N\) of these elements, with \(N > 1\) and the
axis of each element parallel to that of all others. The superscript \(i\) denotes quantities associated with the \(i\)th element. The elements are stacked "end" to "end" along the \(z\)-axis:

\[
z_i^* = z_0^i + (i = 1, 2, ..., N - 1)
\]

The point where the magnetic field is to be determined \((\rho, z)\) is selected to be somewhere between the endpoints of the first \((z_0^1)\) and \(N\)th \((z_N^N)\) elements, which are in turn selected so that

\[
(z - z_0^1) > \rho > (z_N^N - z)
\]

This gives

\[
\frac{1}{2} \{\tanh[D(z - z_0^1)] - \tanh[D(z - z_N^N)]\} \approx 1
\]

and

\[
\frac{1}{2} \left( \frac{z - z_0^1}{\sqrt{\rho^2 + (z - z_0^1)^2}} - \frac{z - z_N^N}{\sqrt{\rho^2 + (z_N^N - z)^2}} \right) \approx 1
\]

Using type II elements and setting

\[
\gamma^i = \gamma = 1 \quad a^i = a \quad D_0^i = D = D \quad (\text{all } i)
\]

The magnetic field due to these current elements is given by

\[
B_\phi = g_2(\rho, z) \sum_{i=1}^{N} f_2^i(\rho, z)
\]  

(29)

which is just

\[
B_\phi = g_2(\rho, z) \{\tanh[D(z - z_0^1)] - \tanh[D(z - z_N^N)]\}
\]  

(30)

or, using the restrictions placed on \(z\), \(z_0^1\) and, \(z_N^N\) above,

\[
B_\phi \approx a \begin{cases} \frac{1}{\rho} & \rho > a \\ \frac{\rho}{a} & \rho < a \end{cases}
\]

(31)

If type I elements are used (with \(a^i = a\) (all \(i\))), then the same total magnetic field is obtained. This is the magnetic field due to an infinitely long current-carrying wire of finite radius \(a\), with a distributed current density along the axis of

\[
J_\phi = \frac{2a}{\mu_0 a^2}
\]

(32)

Thus, stacking identical elements of either type "end" to "end" along a straight line gives the magnetic effect of a long cylinder of finite radius carrying current. The current that flows out of the "end" of one element flows smoothly into the "end" of the next element and so on. The idea behind this approach is to create the magnetic effects of current-carrying flux tubes by stacking these elements, one after another, along model magnetic field lines. The "ends" of elements stacked around curved diverging field lines will not match up exactly and current will "leak" out at the junctions. If type I current elements are used and care is taken to close the circuit, the current will be identically zero away from the elements. This will not be true for type II elements.

The T87 model is the background magnetic field on which the magnetic effects of the model field-aligned current system are superposed. Initially, a model current-carrying flux tube is constructed using current elements. The points at \(z_0\) and \(z_1\) along the \(z\)-axis of each element are positioned on the magnetic field line. To create the magnetic effect of a field-aligned current sheet, this procedure is carried out along a large number of closely spaced field lines. The magnetic effects of the morning sector region I and region II current sheets are modeled by placing the ionospheric ends of these sheets as indicated in Figure 10. By applying symmetry relationships, this pattern is extended to model region I and II currents in the evening and morning sectors in the southern and northern hemispheres. Each region I flux tube

Fig. 10. View from above the north pole of the northern polar region showing a typical locus of points used as footprints of the model night-side region I (marked "I" on the figure) and II (marked "II" on the figure) field-aligned currents.
has a corresponding region II flux tube with an ionospheric footprint in the same magnetic meridian. The two flux tubes are connected in the ionosphere by a meridional current element. The closure of these currents in the magnetosphere is through one of two possible paths, illustrated in Figures 11 and 12. The first closure path (Figure 11) connects the region I and region II flux tubes through a radial current element in the equatorial plane while the second circuit connects the dawn and dusk sector region II currents azimuthally in the equatorial plane and takes the region I currents out to the distant magnetosphere.

In a process similar to one outlined by Kaufmann and Larson [1989], the field lines on which the current elements are centred are retraced a number of times using the modified magnetic field, in an attempt to iterate towards a “field-aligned” configuration. How “field-aligned” the final configuration will be is dependent on how much current is added and how far out in the magnetotail the currents are taken. It is essential that the current configuration be as close to field-aligned as is possible if mapping studies are to be carried out on field lines that are in the vicinity of the current sheets. A typical consequence of this condition not being met well enough is that the field lines of interest will cross a current sheet. If this happens, the integrated effects of the field-aligned currents along such a field line will be far from reasonable.

There are several criteria that could be used in the selection of the constant $\alpha$ (which is different, in general, for each element). For a flux tube constructed with these elements, $\alpha$ could be selected so that the current density in the ionosphere is some desired value. If this value is $J_{\|}$ (in $\mu A/m^2$), and the expressions for the magnetic field due to either type I or type II elements are to give values in nanoteslas with the input parameters in units of Earth radii, then for any element along this model flux tube

$$\alpha = \mu_0 a^2 J_{\|} 10^5 R_E/2 \quad (33)$$

where $a_i$ is the radius of the ionospheric end of the flux tube in Earth radii, $\mu_0 = 4\pi \times 10^{-7}$ and $R_E$ is the radius of the Earth in meters. A second possibility is to select values of $\alpha$ so that the total amount of current flowing in the field-aligned circuit is some desired value. A third possibility is to parameterize the amount of current in the model field-aligned current distribution so that a desired maximum perturbation of the eastward component of the magnetic field in the top-side ionosphere (i.e., at TRIAD altitude) is produced.

For this study, values of the radius $a$ of the elements are chosen so that, at the ionospheric end of the flux tubes, the current sheets have an ionospheric footprint with a north-south extent of roughly 1.5ø. The radius $a$ of the elements increases with increasing distance from the Earth, corresponding to the increasing cross-sectional area of the flux tube. The coefficient $\alpha$, and therefore the amount of current carried by an element, is the same for each element of a model current loop.

A complication arises because of the use of cylindrical current elements and the fact that, in reality, circles in the ionosphere do not map to circles in the magnetosphere. A model flux tube, constructed out of cylindrical elements, will have an increasingly unrealistic cross-section with increasing distance from the Earth. A current sheet constructed from these flux tubes will remain a current sheet; however, the rate at which the current sheet thickens will not be "correct", in the sense that the magnetic flux contained in the current sheet will vary with distance from the Earth. Figure 13 shows a comparison of the thickness of a typical model current sheet with that of a current sheet with the same ionospheric footprint but is also threaded by constant magnetic flux. The model current sheets are always thinner than they

![Fig. 11. One closure path of field-aligned currents used in this study. Here the region I and region II current elements are closed with a north-south current element in the ionosphere and a radial current element in the equatorial plane of the magnetosphere.](image1)

![Fig. 12. One closure path of field-aligned currents used in this study. Here region I and region II current elements joined in the ionosphere with north-south current elements. The dawnside region II current is connected to the duskside region II current azimuthally in the equatorial plane. The region I currents are taken out to the outer magnetosphere.](image2)

![Fig. 13. Current sheet thickness as a function of distance from the Earth. These values are determined along a field line traced using the T87 KP=2 magnetic field model. The field line footprint is in the ionosphere at 69º latitude and MLT=12 hours. The expected thickness, based only on magnetic flux conservation, of a current sheet that is 0.030 $R_E$ thick in the ionosphere and is centred (latitudinally) on this field line is shown by the dotted curve. The dashed curve shows the thickness of a model current sheet. The solid curve shows the ratio of the expected current sheet thickness to the model current sheet thickness.](image3)
would be if the magnetic flux threading them did not vary with distance from the Earth. It would be possible to adjust the flux tube radii so that the magnetic flux threading a model current sheet does not vary with distance from the Earth. In the present work, however, this has not been done for the following two reasons, one of which applies to the region I currents sheet and the other to region II current sheets. First, in the inner magnetosphere, the difference between the model current sheet thickness and the "correct" current sheet thickness is small except near the equatorial plane, where the current is being diverted into transverse closure current. Therefore, this effect is not important for the model region II currents. Second, although the effect is more important for the region I currents (away from the equatorial plane the ratio of the correct to the modeled layer thicknesses can be 1.5 or more), several processes will broaden a current sheet causing it to have a larger ionospheric footprint than magnetic flux conservation alone would indicate. For example, the dawn-dusk electric field will cause charged particles in the PSBL to drift towards the equatorial plane [e.g., Onsager et al., 1991].

RESULTS

The model presented here provides magnetic field values due to nightside region I and II currents on the dawn and dusk sides of the northern and southern hemisphere magnetosphere. The model currents are closed through two different paths, as shown in Figures 11 and 12. In the near-Earth region, currents closed either way produce similar magnetic field perturbations. Here "near-Earth" means Earthward of where field lines, on which model region II currents are placed, intersect the equatorial plane. On the other hand, further out in the magnetotail, currents closed through the two possible paths will produce magnetic perturbations that differ significantly. To produce the results presented below, model region II currents are placed on field lines that cross the equatorial plane roughly 8 Re from the Earth. The regions Earthward and tailward of 8 Re are referred to as the inner and outer magnetosphere, respectively. For both regions, effects of added currents on the magnetic field and on mappings between the ionosphere and the region in question are shown.

The inner magnetosphere results are produced using currents constructed with type I elements and closed as shown in Figure 11. Figure 14 shows magnetic field vectors, due to the model current system, determined on a surface 6 Re from the Zgpm axis. Figures 16 and 17, respectively. Away from the edges of the current sheets and in the near-Earth region, where the azimuthal component of the background magnetic field is small, the peak azimuthal component of the magnetic perturbation due to the field-aligned currents should vary as 1/Rgpm, where Rgpm is the distance from the Zgpm axis. In Figure 18, I show a comparison of the model peak perturbations in the 0300 hours MLT meridian with those expected on the basis of a 1/Rgpm dependence.

I have used this model to investigate the effects of field-aligned currents on mapping between the ionosphere and geosynchronous orbit. Field lines are traced from a point...
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that is 6.63 RE from the Earth's centre. Two ionospheric footprints of this point are determined by tracing field lines toward the ionosphere using both the T87 model and a modification of it that includes field-aligned current effects. The footprint determined from a T87 model tracing will differ, in both longitude and latitude, from the footprint obtained from a tracing produced by the modified T87 model. These differences are determined for points on two curves, one at 0300 hours MLT and the other at 0125 hours MLT. Each curve covers the range from 2 to 5 RE above the equatorial plane (2 < Z_gsm < 5) on a surface 6.63 RE from the Earth's centre. The results for 0300 and 0125 hours MLT are presented in Figures 19 and 20, respectively, and were produced using an amount of field-aligned current that gives an eastward magnetic perturbation of 420 nT at TRIAD altitude.

Together, Figures 16 and 19 illustrate that the longitude shift of the ionospheric footprint of points at geosynchronous orbit is largest for points that are between the region I and

Fig. 16. Magnetic field due to the model field-aligned current system on a surface 6.63 RE from the Earth's centre (geosynchronous orbit distance). For points on the curve at 0300 hours local time from Z_gsm = 2RE to Z_gsm = 5RE, the azimuthal (solid curve) and Z (dashed curve) components of the magnetic field due to the model field-aligned currents are shown. The field-aligned current consists of region I and region II currents with ionospheric footprints as shown in Figure 10. Type I current elements are used. The amount of current in the field-aligned current system gives a 450 nT eastward magnetic field perturbation at TRIAD altitude.

Fig. 17. Same as Figure 16, but values determined on a curve at 0125 hours local time.

Fig. 18. The expected (solid curve) and model (crosses) maximum values of the azimuthal magnetic field due to a model field-aligned current system as a function of distance from the Z_gsm axis. The model current system consists of region I and region II currents with ionospheric footprints as shown in Figure 10. Type I current elements are used. The current system spans 5.5 hours of local time from MLT=0.5 hours to MLT=6 hours. The values shown here are from MLT=3 hours.

Fig. 19. Shift in longitude (solid curve marked φ) and latitude (dashed curve marked θ) of the footprints of field lines traced towards the Earth from points on a curve at geosynchronous distance and 0300 hours local time. The values on the plots are obtained by tracing field lines towards the Earth first using the T87 model and then using the T87 model with field-aligned currents added. The model field-aligned current system used here is the same as that used to produce the results shown in Figure 16.

Fig. 20. Same as Figure 19, but values determined on a curve at 0125 hours local time.
Fig. 21. Maximum westward local time shift of the ionospheric footprint of field lines traced from points at 300 hours local time and on a surface 6.63 $R_E$ from the Earth's centre. The local time shift is plotted as a function of the maximum perturbation of the eastward component of the magnetic field at TRIAD altitude (800 km).

Fig. 22. Vector plot showing the magnetic perturbation due to added field-aligned and closure currents on the surface $X_{gsm} = -15R_E$. The amount of current added gives an eastward perturbation of 125 nT at TRIAD altitude. The region I and II currents are closed in the magnetosphere as shown in Figure 11. The peak magnitude of vectors on this plot is 5.7 nT and all vectors with magnitudes less than 2 nT are set to zero length for clarity.

Fig. 23. Vector plot showing the magnetic perturbation due to added field-aligned and closure currents on the surface $X_{gsm} = -15R_E$. The amount of current added gives an eastward perturbation of 65 nT at TRIAD altitude. The region I and II currents are closed in the magnetosphere as shown in Figure 12. The peak magnitude of vectors on this plot is 3.9 nT and all vectors with magnitudes less than 1 nT are set to zero length for clarity.

II currents. The maximum longitude shift is a function of the amount of current flowing in the added current system (Figure 21).

In the outer magnetosphere, the manner of current closure has a significant effect on the magnetic perturbations produced. Magnetic perturbations due to current systems closed as shown in Figures 11 and 12 are compared in Figures 22 through 25. Figures 22 and 23 are vector plots showing magnetic perturbations due to field-aligned currents. Figures 24 and 25 show the contributions to equatorial $B_z$, due to field-aligned currents, in the noon-midnight meridian and on the line from $Y_{gsm} = -10R_E$ to $Y_{gsm} = 10R_E$ at $X_{gsm} = -15R_E$, respectively.

To illustrate the possible effects of field-aligned currents on mapping between the ionosphere and the outer magnetosphere, field line traces were produced using the T87 $K_p = 2$ model with various amounts of added field-aligned current. Figures 26 and 27 show that a field line (defined here by its ionospheric footprint) is swept towards the flanks of the magnetosphere, due to the magnetic effects of field-aligned currents that are closed as shown in Figure 11. The field line traces shown in both figures are terminated where the field line crosses the equatorial plane.

Fig. 24. Equatorial $B_z$ in noon-midnight meridian due to added field-aligned and closure currents. The current system producing the values shown by the dashed curve gives an eastward magnetic field perturbation of 125 nT at TRIAD altitude and is closed as shown in Figure 11. The solid curve shows values produced by a field-aligned current system that gives an eastward magnetic field perturbation of 62.5 nT at TRIAD altitude and is closed as in Figure 12.

Fig. 25. Equatorial $B_z$ at $X_{gsm} = -15R_E$ due to added field-aligned and closure currents. The current systems producing results shown by the solid and dashed curves are described in the caption for Figure 24.
Fig. 26. Projections in the $XY_{gsm}$ plane of field lines traced using the T87 $Kp = 2$ model modified to include the magnetic effects of field-aligned and closure currents. The five field lines shown here were traced with amounts of field-aligned current added that would produce eastward perturbations of the magnetic field at TRIAD altitude of 0 nT, 62.5 nT, 125 nT, 187.5 nT and 250 nT. Only the field lines corresponding to 0 nT and 250 nT are labeled. 0 nT corresponds to no field-aligned current added. The field lines all have an ionospheric footprint at 68.6° latitude and 0200 hours local time and terminate in the equatorial plane (i.e., at $Z_{gsm} = 0$).

**DISCUSSION**

Two aspects of the results presented above warrant specific discussion. These are the effects of the model field-aligned currents on (1) the magnetic field in the outer magnetosphere and (2) mappings from the ionosphere to the outer magnetosphere. As in the results section, “outer magnetosphere” means tailward of where the model region II currents close in the equatorial plane.

Figures 22 through 25 show the effects of field-aligned currents on the magnetotail magnetic field. Currents closed radially in the equatorial plane (i.e., as in Figure 11) decrease $B_z$ near midnight in the equatorial plane and increase the flare of the CPS magnetic field away from midnight. On the other hand, a current system with region II currents closed azimuthally through a “partial ring current” and region I currents taken out to the distant magnetotail (i.e., as in Figure 12) will increase $B_z$ near midnight in the equatorial plane, but will have little effect on the flare of the CPS magnetic field relative to the noon-midnight meridian. It is often assumed that the field-aligned currents will cause an increase in equatorial $B_z$ near midnight [e.g., Rostoker and Boström, 1976; Donovan et al., 1992]. That field-aligned currents could also cause a decrease in equatorial $B_z$ near midnight is an interesting and unexpected result that could help to explain Fairfield’s [1986] observation that in the magnetotail, $B_z$ is observed to be larger at the flanks than near midnight.

Figures 26 and 27 show that field-aligned currents, closed radially in the equatorial plane, cause CPS field lines equatorward of the region I currents to flare away from midnight. The equatorial crossing point of a field line traced from a specified point in the ionosphere moves away from midnight (to larger $|Y_{gsm}|$) if the magnetic effects of field-aligned currents are added to the background magnetic field model. The shift increases with both amount of added field-aligned current and ionospheric footprint latitude. The field lines are traced using a model with field-aligned currents closed as shown in Figure 11; the same has not been done using currents closed as shown in Figure 12. The vector plots in Figures 22 and 23 clearly show, however, that the effect of field-aligned currents closed as shown in Figure 11 on $B_z$, in the CPS, is much larger than that of currents closed as in Figure 12. Thus the latter field-aligned currents will have much less effect on the flare of CPS field lines.

The mapping results presented here indicate that, due to the magnetic effects of field-aligned currents, field lines threading the CPS might cross the equatorial plane much further from midnight than mappings based on empirical models that do not include these effects would indicate. This is an important result because, for instance, it could shed light on whether or not high-latitude regions of the evening sector auroral oval are connected to the LLBL along magnetic field lines [cf. Rostoker, 1991]. To answer this question convincingly, one way or another, it will be necessary to develop an empirical model that incorporates the global effects of field-aligned currents; however, unless the path of closure of these currents is correct in the model, it is unlikely that the mappings so produced will be any more trustworthy than those available with present models.

This new model is a valuable tool because it allows for the modeling of realistic field-aligned current distributions. The model magnetic field is valid inside of, near and far away from the model field-aligned currents, both near the ionosphere and in the magnetosphere. Furthermore, the modeling technique allows for the closure of field-aligned currents along any specified path. Although I have used model field-aligned currents that form a simple night-side region I and II current system to generate the results presented in this
paper, there is no reason why the magnetic effects of any field-aligned current system could not be modeled with this technique.

The field-aligned current system I have modeled is used to illustrate how these currents might affect both the magnetic field distribution on the night-side and mappings from the ionosphere to the magnetosphere and vice versa. Three other applications of this modeling technique are as follows: (1) model current sheets of finite thickness and width are useful in the interpretation of magnetic field data from low-altitude satellite passes through field-aligned current structures [Fung and Hoffman, 1992]; (2) a model of the magnetic effects of "instantaneous" distributions of field-aligned currents inferred from satellite data [e.g., Marklund et al., 1987] would be useful in an individual event (i.e., substorm) study; (3) it might be possible to use the Vasyliunas equation [Vasyliunas, 1970], and a model of the currents suggested by applying the Vasyliunas equation to an empirical magnetic field model [Stern, 1988], in an iterative fashion, to produce a model magnetic field that better satisfies a simple stress-balance relationship.

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